

Assignment I: MTH 213, Fall 2017

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QUESTION 1. (i) Convince me that $3.128282\dots82\dots$ is a rational number (i.e. write it as a/b where a, b are integers and $b \neq 0$)

(ii) Convince me that $1.817817\dots817\dots$ is a rational number.

(iii) Solve over Z_{15} , $5x = 10$, i.e. $5x \equiv 10 \pmod{15}$.

(iv) Solve over Z , $5x \equiv 10 \pmod{15}$

(v) Solver over Z , $7x \equiv 3 \pmod{14}$

(vi) Solve over Z , $8x \equiv 12 \pmod{20}$

(vii) Find $\gcd(286, 124)$

(viii) Find $\gcd(786, 348)$

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(i) 3.1282828

Hanin Alrais/ Solution

$$x = 3.128\overline{28}$$

$$100x = 312.828$$

$$100x - x = 309.7$$

$$99x = \frac{3097}{10}$$

$$990x = 3097$$

$$x = \frac{3097}{990} \begin{matrix} \leftarrow a \\ \leftarrow b \end{matrix}$$

(ii) 1.817817

$$x = 1.\overline{817}$$

$$1000x = 1,817.817817$$

$$1000x - x = 1816$$

$$999x = 1816$$

$$x = \frac{1816}{999} \begin{matrix} \leftarrow a \\ \leftarrow b \end{matrix}$$

(iii) $5x \equiv 10 \pmod{15}$

$$\gcd(5, 15) = 5 \quad \therefore 5 \text{ solutions}$$

Check $5|10$? Yes.

$$\text{Solution Set } \{2, 5, 8, 11, 14\}$$

(iv) solve over \mathbb{Z} ,

$$5x \equiv 10 \pmod{15}$$

$$\text{solve over } \mathbb{Z}_{15} = \{2, 5, 8, 11, 14\}$$

$$\text{sol. set } \left\{ \begin{matrix} 15n + 2, 15m + 5, \\ 15k + 8, 15j + 11, \\ 15q + 14 \end{matrix} \right\}$$

(v) Solve over \mathbb{Z} ,
 $7x \equiv 3 \pmod{14}$

$$\gcd(7, 14) = 7$$

$$\text{solution} = \{ \}$$

(vi) Solve over \mathbb{Z} ,

$$8x \equiv 12 \pmod{20}$$

$$\gcd(8, 20) = 4$$

Check $4 \mid 12$? Yes

solve over \mathbb{Z}_{20}

Sol. set $\{4, 7, 9\}$

$$\text{over } \mathbb{Z} = \{20n + 4, 20m + 7, 20k + 9, 20j + 12\}$$

(vii) Find $\gcd(286, 124) = 2$

$$\begin{array}{r} 2 \\ 124 \overline{) 286} \\ \underline{-248} \\ 38 \end{array}$$

$$\begin{array}{r} 3 \\ 38 \overline{) 124} \\ \underline{-114} \\ 10 \end{array}$$

$$\begin{array}{r} 3 \\ 10 \overline{) 38} \\ \underline{-30} \\ 8 \end{array}$$

$$\begin{array}{r} 1 \\ 8 \overline{) 10} \\ \underline{-8} \\ 2 \end{array}$$

$$\begin{array}{r} 4 \\ 2 \overline{) 8} \\ \underline{-8} \\ 0 \end{array}$$

(viii) Find $\gcd(786, 348) = 6$

$$\begin{array}{r} 2 \\ 348 \overline{) 786} \\ \underline{-696} \\ 90 \end{array}$$

$$\begin{array}{r} 3 \\ 90 \overline{) 348} \\ \underline{-270} \\ 78 \end{array}$$

$$\begin{array}{r} 1 \\ 78 \overline{) 90} \\ \underline{-78} \\ 12 \end{array}$$

$$\begin{array}{r} 6 \\ 12 \overline{) 78} \\ \underline{-72} \\ 6 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 12} \\ \underline{-12} \\ 0 \end{array}$$

Hanin just told me
the right solution
over \mathbb{Z}_{20} is $\{4, 9, 14, 19\}$
So over \mathbb{Z} is
 $\{20n + 4, 20m + 9, 20k + 14, 20j + 19\}$